

# Homework Assignment #9

MATH 251: Ordinary and Partial Differential Equations

Due Friday, November 9, 2012

In order to get full credit on any of the questions, you are required to **show your work!**

1 - 4 Determine the type and stability of the critical point at  $(0,0)$  of each system below.

1.

$$\mathbf{x}' = \begin{bmatrix} 2 & 7 \\ -5 & -10 \end{bmatrix} \mathbf{x}.$$

2.

$$\mathbf{x}' = \begin{bmatrix} -3 & 6 \\ -3 & 3 \end{bmatrix} \mathbf{x}.$$

3.

$$\mathbf{x}' = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} \mathbf{x}.$$

4.

$$\mathbf{x}' = \begin{bmatrix} 6 & -5 \\ 0 & 6 \end{bmatrix} \mathbf{x}.$$

5. (i) For what value(s) of  $b$  will the system below have an improper node at  $(0,0)$ ? (ii) For what value(s) of  $b$  will the system below have a spiral point at  $(0,0)$ ?

$$\mathbf{x}' = \begin{bmatrix} 5 & b \\ 2 & -1 \end{bmatrix} \mathbf{x}.$$

6 - 7 Find the critical point of each nonhomogeneous linear system given. Then determine the type and stability of the critical point.

6.

$$\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ 6 & -3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} -6 \\ 24 \end{bmatrix}.$$

7.

$$\mathbf{x}' = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 12 \\ -3 \end{bmatrix}.$$

8 - 9 Find all the critical point(s) of each nonlinear system given. Then determine the type and stability of each critical point.

8.

$$\begin{aligned}x' &= xy + 3y \\y' &= xy - 3x\end{aligned}$$

9.

$$\begin{aligned}x' &= x^2y + 3xy - 10y \\y' &= xy - 4x\end{aligned}$$