Math 170-01 Calculus I
Final Exam Review Sheet
May 11, 2015 at 8:00 am

It is highly recommended that you work on as many of these problems as possible. This is not an exhaustive list of the types of questions you might see. You should also review previous in class exams, review sheets, and homework problems.

1. Find the following limits:

   (a) $\lim_{x\to -1} \dfrac{x^3 - x^2}{x-1}$

   (b) $\lim_{x\to 2^-} \dfrac{x+2}{x-2}$

   (c) $\lim_{x\to\infty} \dfrac{2x^2-6}{x^2+5x}$

   (d) $\lim_{x\to\infty} \dfrac{x^3-x^2+10}{3x^2-4x}$

   (e) $\lim_{x\to 0} \dfrac{x\sin(x)}{1-\cos(x)}$

   (f) $\lim_{x\to 0} \ln(\sin(2x)) - \ln(\tan(x))$

   (g) $\lim_{x\to 0^+} \dfrac{\cot(x)}{\ln(x)}$

   (h) $\lim_{x\to\infty} x\sin(\pi/x)$

2. Find any values of x (if they exist) where the function $f(x)$ is not continuous:

   (a) $f(x) = \dfrac{4x+1}{x^2-1}$

   (b) $f(x) = |x^2+3|$

   (c) $f(x) = \dfrac{x+5}{|x^2+5x|}$

   (d) $f(x) = e^{\ln(x)}$

3. Use the limit definition of the derivative to find the slope of the tangent line to the graph of $f(x) = x^2 + 1$ at a general x value. Then, use it to find the slope of the tangent line to the graph of f at $x = 4$.

4. Find $f'(x)$ for each of the following:

   (a) $f(x) = \sin^3(x) + 4\cos(x)$

   (b) $f(x) = \sqrt{5x-2}(x+3)^2$

   (c) $f(x) = \left(\dfrac{4x-2}{x^3}\right)^2$

   (d) $f(x) = \tan^3(x^4)$

   (e) $f(x) = \sqrt[3]{5+\sqrt{x}}$

   (f) $f(x) = 2xe^{\sqrt{x}}$

   (g) $f(x) = \cos^3\left(\dfrac{x}{x+1}\right)$

   (h) $f(x) = \dfrac{6}{1+3e^x}$

   (i) $f(x) = \cos^{-1} x^2$

   (j) $f(x) = \sqrt[3]{\ln(x)+1}$

5. Find the equation of the tangent line to the graph of $y = \ln(2-x^2)$ at $x = 2$.

6. Find y' when

   (a) $x^3 + xy + y^3 = x$

   (b) $3xy^2 - 6x + 3y^3 = 9$

7. Find the derivative of $y = \dfrac{x^3}{\sqrt{x^2+3}}$ using logarithmic differentiation.

8. Use an appropriate local linearization to approximate $(1.97)^3$.

9. A rocket, rising vertically, is tracked by a radar station that is on the ground 5 miles from the launchpad. How fast is the rocket rising when it is 4 miles high and its distance from the radar station is increasing at a rate of 2000 miles per hour?

10. For each of the following functions, find (a) the intervals on which the function is increasing or decreasing and (b) the intervals on which the function is concave up or concave down.

    (a) $f(x) = \dfrac{x-2}{(x^2-x+1)^2}$

    (b) $f(x) = x^3 \ln(x)$

11. Sketch a graph of the following functions by calculating critical values, inflection points, intercepts, intervals of increasing and decreasing, intervals of concavity, asymptotes, etc.

    (a) $f(x) = x^2 - 3x - 4$

    (b) $f(x) = \dfrac{2x-6}{4-x}$

12. Sketch the graph of a function $f(x)$ where

$$f(0) = -1, \qquad f'(0) = 0, \qquad f'(-1) < 0, \qquad \lim_{x\to\infty} f(x) = \infty, \qquad \lim_{x\to-\infty} f(x) = 3$$

13. A rectangular area of 3200 square feet is to be fenced off. Two opposite sides will use fencing costing \$1 per foot, and the remaining sides will use fencing costing \$2 per foot. Find the dimensions of the rectangle with the lowest possible cost.

14. Verify that the hypotheses of Rolle's Theorem are satisfied on the interval $[-1, 3]$ for the function $f(x) = \ln(4 + 2x - x^2)$, and find all values of c in that interval that satisfy the conclusion of the theorem.

15. Find the integrals:

    (a) $\int (4x^3 - 6x + 8)\, dx$

    (b) $\int_1^4 \dfrac{4}{x^2}\, dx$

    (c) $\int_1^{\sqrt{2}} xe^{-x^2}\, dx$

    (d) $\int_{-1}^{1} \dfrac{dx}{1+x^2}$

    (e) $\int \tan(2\theta)\, d\theta$

    (f) $\int_{1/2}^{1} \dfrac{1}{2x}\, dx$

    (g) $\int_{-2}^{-1} \dfrac{x}{(x^2+2)^3}\, dx$

    (h) $\int_0^2 |2x-3|\, dx$

    (i) $\int x(1+x^3)\, dx$

16. Let $F(x) = \int_1^x (t^3 - \sin^2(t))\, dt$. Use the Fundamental Theorem of Calculus to find $F'(x)$.