Exam 1 Review Solutions
Math 170 - Calculus I

Note: For most questions below I have only provided the final answer. You will likely need to show more work than I have for many of these problems. See/ask me right away if you have any questions about how I arrived at a solution.

1. Find $(f \circ g)(x)$ and $(g \circ f)(x)$ if $f(x) = 3x$ and $g(x) = 2x + 5$

$$(f \circ g)(x) = f(g(x)) = f(2x+5) = 3(2x+5) = 6x + 15$$
$$(g \circ g)(x) = g(f(x)) = g(3x) = 2(3x) + 5 = 6x + 5$$

2. Decide if the function $f(x) = 6x - 8$ has an inverse and, if so, find it. Verify that you have found the inverse of $f(x)$.

$$f^{-1}(x) = \frac{x+8}{6}$$
To verify, show that
$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

3. Simplify (write as a single expression): $6\log_4 3 - 4\log_4 5x$

$$6\log_4 3 - 4\log_4 5x = \log_4(3^6) - \log_4((5x)^4) = \log_4\left(\frac{3^6}{(5x)^4}\right)$$

4. Expand $\ln\left(\frac{5x^3\sin(x+1)}{6\sqrt{x+3}}\right)$ as a sum and difference of logarithms.

$$\ln\left(\frac{5x^3\sin(x+1)}{6\sqrt{x+3}}\right) = \ln(5) + \ln(x^3) + \ln(\sin(x+1)) - \ln(6) - \ln(\sqrt{x+3})$$
$$= \ln(5) + 3\ln(x) + \ln(\sin(x+1)) - \ln(6) - \frac{1}{2}\ln(x+3)$$

5. Solve the equations for x:

(a) $e^x - 2xe^x = 0$

$$e^x(1-2x) = 0$$
Since $e^x \neq 0$ for any x, the only way this equation is true is if $1 - 2x = 0$, so if $x = \frac{1}{2}$

(b) $\ln(1/x) = 10$

$$\frac{1}{x} = e^{10}$$
$$x = \frac{1}{e^{10}}$$

6. Find the vertical and horizontal asymptotes, if any, of the function $f(x) = \frac{x^2 - 3x}{2x - 2}$

The degree of the numerator is larger than the degree of the denominator, so the limit at $\pm\infty$ would be infinite. Thus, there are no horizontal asymptotes. A vertical asymptote would occur where the function is not defined, so at $x = 1$. Since we can't reduce the function to one that is defined at $x = 1$, we do have a vertical asymptote there.

7. Find the limits:

(a) $\lim_{x\to 3} \frac{4x-12}{x^2-x-6}$

$$\lim_{x\to 3} \frac{4x-12}{x^2-x-6} = \lim_{x\to 3} \frac{4(x-3)}{(x-3)(x+2)} = \lim_{x\to 3} \frac{4}{x+2} = \frac{4}{5}$$

(b) $\lim_{x\to 0^+} \frac{1}{x^2} + 3$

$$\lim_{x\to 0^+} \frac{1}{x^2} + 3 = \infty$$

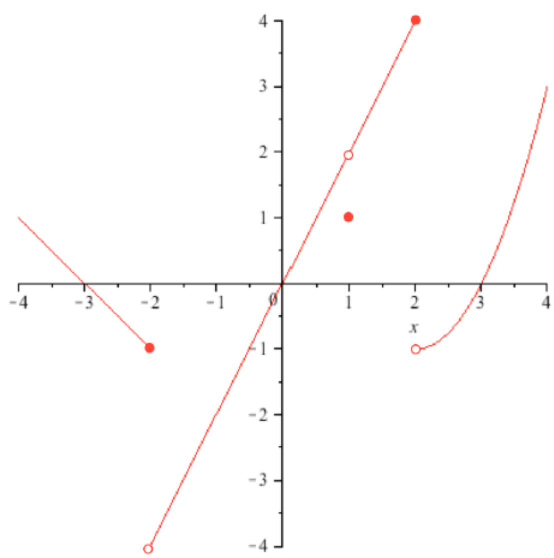
(c) $\lim_{x\to\infty} \frac{7x^2+2x-3}{4x^2+5x+11}$

$$\lim_{x\to\infty} \frac{7x^2+2x-3}{4x^2+5x+11} = \frac{7}{4}$$

(d) $\lim_{x\to 0} \frac{\sin(4x)}{5x}$

$$\lim_{x\to 0} \frac{\sin(4x)}{5x} = \lim_{x\to 0} \frac{4\sin(4x)}{5(4x)} = \frac{4}{5}\lim_{x\to 0} \frac{\sin(4x)}{4x} = \frac{4}{5}$$

8. Refer to the graph of the function $f(x)$ below:



(a) Find $f(-2), f(0), f(1),$ and $f(4)$

$$-1, 0, 1, \text{and } 3, \text{respectively}$$

(b) Find $\lim_{x\to -2} f(x)$, $\lim_{x\to -2^+} f(x)$, $\lim_{x\to 1} f(x)$, $\lim_{x\to 0} f(x)$, and $\lim_{x\to -2^-} f(x)$

$$\text{DNE}, -4, 2, 0, \text{and } -1, \text{respectively}$$

(c) Find any points x where $f(x)$ is discontinuous, and classify he type of discontinuity

$x = -2$ and $x = 2$ are jump discontinuities, $x = 1$ is a removable discontinuity, and there may be an infinite discontinuity near $x = 4$.

9. Where is the function $f(x) = \dfrac{x^2 - 9}{x^2 + 8x + 15}$ discontinuous?

Wherever its denominator is zero, so we have to solve

$$x^2 + 8x + 15 = 0$$
$$(x + 5)(x + 3)$$

So f has discontinuities at $x = -5$ and $x = -3$.

10. Which of the following functions are continuous everywhere?

    (a) $sin(x^3 + 2x)$

    (b) $3x^2 + \tan(x)$

    (c) $\sqrt{5x + 3}$

The first function is the only one. It's a composition of two functions that are continuous everywhere (for all real numbers). The second has a discontinuity where $\tan(x)$ is undefined. The third function is not defined for any values of x less than -3/5.