Exam III Review
Math 170 Calculus I

Exam III will be based on sections 4.1, 4.2, 4.3, 4.4, 4.5, 4.8, 5.1, 5.2, and 5.3

You will be expected to know:

- how to find the x and y intercepts of a function
- what the critical points of a function are, and how to find them
- what the inflection points of a function are, and how to find them
- how to determine the intervals where a function is increasing and decreasing
- how to determine the intervals where a function is concave up and concave down
- the first and second derivative tests
- how to find vertical and horizontal asymptotes of a function, if they exist
- the Extreme Value Theorem
- how to find relative and/or absolution minimum and maximum values of a function
- how to use all of the above to graph polynomial and rational equations
- how to solve applied max and min problems
- the hypotheses and conclusion of Rolle's Theorem
- the hypotheses and conclusion of the Mean Value Theorem
- how to find simple indefinite and definite integrals using the table and properties in section 5.2
- how to evaluate indefinite and definite integrals using substitution

Some sample problems

1. Sketch the graph of a function $f(x)$ where
   (a) $f(-2) = -1$, $f(0) = 0$, $f(2) = 1$, $\lim\limits_{x\to-\infty} f(x) = 0$, and $\lim\limits_{x\to\infty} f(x) = 0$
   (b) $f(0) = -1$, $f'(0) = 0$, $f'(-1) < 0$, $\lim_{x\to\infty} f(x) = \infty$, and $\lim_{x\to-\infty} f(x) = 3$

2. Find the absolute maximum and minimum of $f(x) = 15x^4 - 15x^2 + 31$ on $[-1, 2]$.

3. Sketch a graph of the following functions by calculating critical values, inflection points, intercepts, intervals of increasing and decreasing, intervals of concavity, asymptotes, etc.
   (a) $f(x) = x^2 - 3x - 4$
   (b) $f(x) = \dfrac{2x-6}{4-x}$

4. A cylindrical can is being designed to hold 100 cm^3 of oil. The cost of the can depends only on its surface area. Find the dimensions (height and radius) of the can that will minimize the cost of production. (Hint: You need to find an equation for the surface area of a cylinder. The volume of a cylinder with radius r and height h is $V = \pi r^2 h$; the area of a circle of radius r is πr^2, and the perimeter of circle of radius r is $2\pi r$.)

5. Let $f(x) = \frac{1}{2}x - \sqrt{x}$. Verify that the hypotheses of Rolle's Theorem are satisfied on the interval $[0, 4]$, and find any values of c predicted by the theorem.

6. Let $f(x) = x^3 + x - 4$. Verify that the hypotheses of the Mean Value Theorem are satisfied on the interval $[-1, 2]$, and find any values of c predicted by the theorem.

7. Find the integrals
   (a) $\int 4x^2 - 5x^3 + 1 \, dx$
   (b) $\int \sin(x) - \cos(x) \, dx$
   (c) $\int x^4 + x^{-4} \, dx$
   (d) $\int \dfrac{4x^8 - 2x^4 + 13x^2}{x^3} \, dx$
   (e) $\int \, dx$ (this is not a typo!)
   (f) $\int (x + \sqrt[3]{x})(2 - x^2) \, dx$
   (g) $4\int \sec^2(x) \, dx$
   (h) $\int \dfrac{5 - 3\sin^2(x)}{\sin^2(x)} \, dx$
   (i) $\int \dfrac{4}{x^2} \, dx$
   (j) $\int \dfrac{dx}{1+x^2}$
   (k) $\int \tan(2\theta) \, d\theta$
   (l) $\int x(1 + x^3) \, dx$

8. Find the solution $y(x)$ of the initial value problems:
   (a) $y' = 3x^2 - 4, \quad y(0) = 2$
   (b) $y' = 4x^3 - 9 + 2\sin(x) + 7e^x, \quad y(0) = 15$