Exam II Review
Math 170 Calculus I

Exam II will be based on material from Chapters 2 and 3.

You will be expected to know:

- how to find the tangent line to a function at a point
- the relationship between the tangent line of a function and the derivative
- how to find the derivative of a function using the limit definition
- the power rule
- the product rule
- the quotient rule
- the chain rule
- the derivative of the trig functions $\sin(x)$, $\cos(x)$, and $\tan(x)$
- how to find the derivative of a function implicitly
- what logarithmic differentiation is, and how and when to apply it
- the derivatives of $\ln(x)$ and e^x
- the derivatives of $\ln(f(x))$ and $e^{f(x)}$
- the derivative of $\sin^{-1}(x)$, $\cos^{-1}(x)$, and $\tan^{-1}(x)$
- how to recognize an indeterminate form
- L'Hôpital's rule, and how and when to apply it

Some sample problems

1. Find a point on the graph of $y = x^2$ where the tangent line to y is parallel to the line $y = 6x - 4$.

2. Find $f'(x)$ using the limit definition of the derivative when $f(x) = 3x^2 - 1$. Check your answer using the power rule.

3. Let $f(x) = 4x^2 + 2x - 3$. Find the intervals, if any, where $f(x)$ is increasing. Find the intervals, if any, where $f(x)$ is decreasing.

4. Find the $\frac{dy}{dx}$ (aka y'), using any method you prefer.

   (a) $y = \left(\dfrac{x^2 - 2}{2x^2 + 1}\right)^3$

   (b) $y = \sin(x^3 + 2)$

   (c) $y = \dfrac{e^x}{\ln(x)}$

   (d) $3xy^2 + 6y - 4x^3 = 10$

   (e) $y = e^x \sin^{-1}(x)$

   (f) $y = \left(\dfrac{\sin(x)}{\cos(x)}\right)^4$

   (g) $y = \ln(\tan(x))$

   (h) $y = \dfrac{-2x^3 - 5x}{8x^2 - 10x}$

   (i) $y = e^{\sqrt{1-3x^2}}$

   (j) $y = \sin^{-1}(x) + \cos^{-1}(x)$

5. Find y'' if $x^2 + y^2 = 50$. Simplify your answer as much as possible.

6. Find y' using logarithmic differentiation if $y = \left(\frac{x^2-5}{3x^4+6}\right)(x^3 + 1)(3x^4 - 5x^2 + 2x)$.

7. Two parallel sides of a rectangle are being lengthened at the rate of 2 in/sec, while the other two sides are shortened in such a way that the figure remains a rectangle with constant area 50 in^2. What is the rate of change of the perimeter of the rectangle when the length of an increasing side is 5 in? Is the perimeter increasing or decreasing? **NOTE:** There will be at least one related rates problem on the exam - it may not be similar to this one, but the more you practice, the better you'll understand the overall strategy.

8. Find the limits:

   (a) $\displaystyle\lim_{x\to+\infty} \frac{x^{60}}{e^x}$ (b) $\displaystyle\lim_{x\to+\infty} \frac{5x^3 - 4x + 3}{2x^2 - 1}$ (c) $\displaystyle\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\sin(x)}\right)$